


Provided for non-commercial research and education use.  
Not for reproduction, distribution or commercial use.

	<h1>Economic Modelling</h1>	
Volume 29, Issue 2 March 2012	<b>CONTENTS</b>	
<b>Co-Editors:</b> STEPHEN HALL PETER PAULY	Is East Asia an optimum currency area? 87 <i>G.H.Y. Lee, M. Azali</i>	
<b>Associate Editors:</b> R. BARRELL R.C. BRYANT W.W. CHAREMZA J. FEDDERKE M. FUNKE B.S. GANGNES S. HOLLY M. JUILLARD L.R. KLEIN L.J. LAU D. LAXTON J.-L. LIN P. McADAM M. MOORE F. MOSCONE P. NIJKAMP D. OSBORN G. TAVLAS A. WELFE S. WREN-LEWIS	The prospects of a monetary union in East Asia 96 <i>G.H.Y. Lee, S.G.M. Koh</i>	
	Worker remittances, migration, accumulation and growth in poor developing countries: Survey and analysis of direct and indirect effects 103 <i>T.H.W. Ziesemer</i>	
	Modeling the dependence structure between default risk premium, equity return volatility and the jump risk: Evidence from a financial crisis 119 <i>N. Naifar</i>	
	Bank regulation and supervision and its welfare implications 132 <i>M. Kilinc, B. Neyapti</i>	
	Asset arbitrage and the price of oil 142 <i>V. Arora, R. Tyers</i>	
	Currency crisis transmission through international trade 151 <i>J.I. Haider</i>	
	Empirical determination of aggregate demand and supply curves: The example of the RWI Business Cycle Model 158 <i>U. Heilemann, H. Findeis</i>	
	The impact of fiscal policies on agricultural household decisions 166 <i>T. Glauken, T. Hertzfeld, J.-P. Loy, S. Renner, H. Hackmann</i>	
	On ridge estimators for the negative binomial regression model 178 <i>K. Másson</i>	
ISSN 0264-9993	(contents continued on outside back cover)	

This article appeared in a journal published by Elsevier. The attached copy is furnished to the author for internal non-commercial research and education use, including for instruction at the authors institution and sharing with colleagues.

Other uses, including reproduction and distribution, or selling or licensing copies, or posting to personal, institutional or third party websites are prohibited.

In most cases authors are permitted to post their version of the article (e.g. in Word or Tex form) to their personal website or institutional repository. Authors requiring further information regarding Elsevier's archiving and manuscript policies are encouraged to visit:

<http://www.elsevier.com/authorsrights>



Contents lists available at SciVerse ScienceDirect

## Economic Modelling

journal homepage: [www.elsevier.com/locate/ecmod](http://www.elsevier.com/locate/ecmod)

## Dynamic general equilibrium model with uncertainty: Uncertainty regarding the future path of the economy

Stephen Pratt<sup>a,\*</sup>, Adam Blake<sup>b,1</sup>, Peter Swann<sup>c</sup><sup>a</sup> School of Hotel & Tourism Management, The Hong Kong Polytechnic University, 17 Science Museum Road, TST East, Kowloon, Hong Kong<sup>b</sup> Services Management School, Bournemouth University, Fern Barrow, Talbot Campus, Poole, Dorset BH12 5BB, UK<sup>c</sup> Nottingham University Business School, Wollaton Road, Nottingham NG8 1BB, UK

## ARTICLE INFO

## Article history:

Accepted 20 February 2013

Available online xxxx

## JEL classification:

C68

C63

## Keywords:

Computable general equilibrium model

Uncertainty

Risk

Tourism

## ABSTRACT

This paper develops a new method for incorporating uncertainty within a computable general equilibrium (CGE) model. The method involves incorporating uncertainty into the model by formulating different states of the world or paths that the economy may take. The risk then is that on one or more of the paths, there may be an external demand shock, for example, an exogenous shock in tourism demand. The multi-sector forward-looking CGE model with risk shows the impact of uncertainty on the economy and how households and industry respond to the presence of uncertainty. The results show that, where there is an asymmetric shock, the possibility of a future tourism demand shock creates a welfare loss. The welfare gains along the non-shocked path are a result of household's risk aversion and their substituting resources away from the shocked path. The difference in the monetary values of the welfare on the different paths can be interpreted as the 'price' of the risk. It is the price households would pay to remove the possibility of the tourism shock. Therefore, this research was able to quantify the monetary value of the risk. This method can be used in scenario modelling for other adverse contingent events, such as the uncertainty of climate change impacts, and agriculture production risks.

© 2013 Elsevier B.V. All rights reserved.

## 1. Introduction

The concept of risk has been examined from many different disciplines: from an economic perspective (Anscombe & Aumann, 1963; Arrow, 1965; Kahneman & Tversky, 1979; Pratt, 1964; Rothschild & Stiglitz, 1970, 1971; von Neumann & Morgenstern, 1944), from a sociological perspective (Finucane & Holup, 2005; Slovic, 1986, 1987; Slovic et al., 1985), from a financial perspective (Bluhm et al., 2002) and from a technical perspective (Kammen & Hassenzahl, 1962). Risk is a complex construct. Risk has been defined in many different ways. One frequently cited definition of risk is that of Knight (1921). He defines risk as "measurable uncertainty". Denenberg et al. (1974) simply define risk as "uncertainty of loss". There can be many types of losses as well. Denenberg et al. take a very narrow view of risk defining loss as the loss of wealth or profit. Loss could be a loss of satisfaction/happiness or utility as in the economic meaning of utility. Thus, a loss of utility could involve a financial loss or may involve dissatisfaction or simply just the loss of happiness. This can be measured as a loss in economic welfare.

The CGE class of models is empirically estimated by Arrow and Debreu (1954) using general equilibrium models with empirical data. CGE models were developed in the early 1960s to solve for both market prices and quantities simultaneously, thus simulating the working of a competitive market economy. A CGE model attempts to model the whole economy and the relationships between the economic agents in it. The model solves for a set of prices, including production prices, factor prices, and exchange rate and levels of production that clear all markets. The result is that, following the neo-classical assumption, producers maximise profits, which are the difference between revenue earned and the cost of factors and intermediate inputs. Commodity market demands depend on all prices and satisfy Walras's law. That is, at any set of prices, the total value of consumer expenditures equals consumer incomes. Technology is described by constant returns to scale production functions. Producers maximise profits. The zero homogeneity of demand functions and the linear homogeneity of profits in prices (i.e. doubling all prices double money profits) imply that only relative prices are of any significance in such a model. The absolute price level has no impact on the equilibrium outcome (Rutherford & Paltsev, 1999).

In conventional forward-looking dynamic CGE models, economic agents are endowed with perfect foresight, so both consumers and firms anticipate any exogenous shocks and adjust their maximising behaviour from the first time period onwards. Perfect foresight then would appear to negate any uncertain response to a shock. Taking a

\* Corresponding author. Tel.: +852 3400 2268; fax: +852 2362 9362.

E-mail addresses: [stephen.pratt@polyu.edu.hk](mailto:stephen.pratt@polyu.edu.hk) (S. Pratt),[ablake@bournemouth.ac.uk](mailto:ablake@bournemouth.ac.uk) (A. Blake), [p.swann@nottingham.ac.uk](mailto:p.swann@nottingham.ac.uk) (P. Swann).<sup>1</sup> Tel.: +44 1202 961521; fax: +44 1202 515707.

simple model with Ramsey economic growth dynamics, this paper illustrates a framework that incorporates uncertainty by allowing alternative future time paths resulting in uncertainty in the model. When an adverse shock occurs on one of the paths, this uncertainty is realised as a risk.

The next section outlines the way risk is treated in standard CGE model, whether they are comparative static models, or dynamic model (both dynamic recursive or forward-looking). Section 3 assesses previous research that has attempted to incorporate risk and uncertainty into CGE models. Section 4 in this paper conceptually describes the explicit treatment of risk in a CGE model involving the creation of multiple future paths for the model, where agents are able to predict each path and make decisions, given an element of risk aversion, in the presence of this uncertainty. Section 5 takes the conceptualisation of the uncertainty explained in Section 4 and applies it to a stylised benchmark economy to show the impact of the uncertainty on the economy. The implications the uncertainty has for the behaviour of the different economic agents (households, tourists, government, and industry) are highlighted. Section 6 concludes and suggests areas for further research using this uncertainty framework.

## 2. Risk in standard CGE models: behaviour of microeconomic agents

The section outlines the basic characteristics of the different types of CGE models: the static model, the dynamic recursive model, the single sector dynamic forward-looking model and the multi-sector dynamic forward-looking model. For each type of model, the implicit assumptions of risk are outlined.

### 2.1. Risk in a comparative static CGE model

The comparative static (within period) CGE model follows the interactions and relationships of a market economy and solves for a set of prices including production prices, factor prices and exchange rate and levels of production that clear all markets. Equilibrium in this model is characterised by a set of prices and levels of production in each industry such that the market demand equals supply for all commodities. Since producers are assumed to maximise profits, and production exhibits constant returns to scale, this implies that no activity (or cost-minimising technique for production functions) does any better than break even at the equilibrium prices. Demand for and supply of goods and services readjust until all excess demands and excess supplies are eliminated through changes in prices. The production function is specified in terms of labour and capital and the amount of each type of these inputs employed by a producer in a particular sector is based on the sector specific production technology and input prices. Perfectly competitive markets operate to determine these equilibrium prices. Additionally, in equilibrium, no sector earns above-normal profits, markets clear for all factors and products, and, in an open economy, the value of imports for intermediate use and final demand equals the value of export earnings. The microeconomic underpinnings of economic agents in a CGE framework follow the traditional neoclassical approach. Agents have rational preferences among outcomes that can be identified and associated with a value. Individuals exhibit maximising behaviour and act independently on the basis of full and relevant information.

In terms of implicit risk, the return on capital captures all the inherent risk associated with the investment and owners of capital are paid an appropriate return, given the level of risk. Elasticities capture the trade-off between the choice of various products and of the inherent risk associated with the curvature of the utility functions. In such models, risk or uncertainty is not explicitly factored into the model.

### 2.2. Risk in a dynamic recursive CGE model

A dynamic recursive model is backward-looking by nature: what happens in future periods does not affect the current year's equilibrium. The model is solved year by year without having to solve the whole study period at once. Agents in these models exhibit myopic behaviour. These sequential dynamic models are basically a series of static CGE models that are linked between periods by behavioural equations for endogenous variables and by updating procedures for exogenous variables. Capital stock is updated endogenously with a capital accumulation equation, whereas exogenous variables such as total labour supply are updated between periods. This process can be seen in Fig. 2. The intra-temporal model is represented by the circular flow diagram with-in the black ovals in Fig. 1. The updating of the exogenous variables flow chronologically from left to right, that is, the within period model solves and then advances to the next time period.

The models are linked together by the savings/investment rule. However, other research has shown that the savings/investment rule can determine to a large extent, the results of the model. The concept of risk in the dynamic recursive model is the same as the treatment of risk in a static model. The 'new' elements in the dynamic recursive model are deterministic in nature and again, the risk is implicitly modelled through the interest rate and in the elasticities.

### 2.3. Risk in a single-sector dynamic forward-looking CGE model

The dynamic forward-looking computable general equilibrium model assumes that consumers' and producers' behaviour is derived from both intra- and intertemporal optimization. These models incorporate some form of life-cycle behaviour. The household maximises an additive separable time-invariant intertemporal utility function, while the producer's optimal behaviour is determined by the maximisation of the market value of the firm or by the maximisation of the present discounted value of net cash flows. The market value of the firm is usually represented as the present discounted value of the future stream of dividends. The model is based upon the perfect foresight hypothesis and describes the transition path to the new equilibrium point. Households and firms make optimal choices given their intertemporal budget constraints. Households maximise the present value of their lifetime utility and firms maximise the value of their profits. In every period, prices adjust to guarantee equilibrium in the model so that demand equals supply. These types of model were first developed by Ramsey (1928), Cass (1965) and Koopmans (1965) (see Barro and Sala-i-Martin, 1995).

As with the comparative static model, demand for and supply of goods and services re-adjust until all excess demands and excess supplies are eliminated through changes in prices. Perfectly competitive markets operate to determine these equilibrium prices. Additionally, in equilibrium, no sector earns above-normal profits, markets clear for all factors and products. Equilibrium is met for each product in each time period by allowing prices to clear markets. The CPI is often adopted as the numeraire, as in this study. Hence all price changes mentioned in the Results section should be interpreted as changes relative to the price of consumer goods (Dixon et al., 2005).

In contrast to the dynamic recursive model, the dynamic forward-looking model does not have a rule that links one time period to the next but capital is accumulated in each future time period (represented by the orange links between the intra-temporal models in Fig. 3). Further, firms maximise the net present value of their profits and consumers maximise their net present value of their utility. They have model-consistent rational expectations about future time periods. Decisions made in period,  $t$  (and subsequent time periods) take into consideration events that occur in future time periods. Economic agents can adjust to shocks before they occur. As represented in Fig. 3, the expectations are made for each time period considering what has happened before and what will happen after the current time period so in period  $t = 2$ , the representative consumer optimises

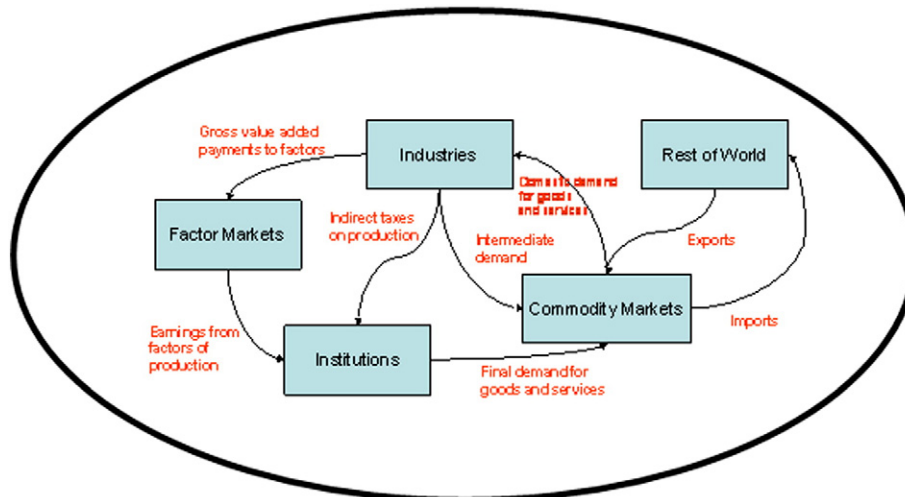


Fig. 1. Comparative static model depicted as a circular flow of income.

their utility given what has happened in period  $t = 1$  and given what will happen in the future time periods.

Differing from the dynamic forward-looking infinite horizon model, the finite horizon model is solved for a certain number of time periods after which the terminal condition then is operationalized (Rutherford, 2004, 2005). This method gives the model tractability. Fig. 4 depicts this graphically so that the model solves up until  $t = n$  at which the terminal condition is enacted.

For both the single-sector dynamic forward-looking model with the infinite and finite horizon versions, risk is implicit as it is with the static and dynamic recursive models.

#### 2.4. Risk in a multi-sector dynamic forward-looking CGE model

This section explains the difference between the single-sector and multiple-sector dynamic intertemporal model and the implications this has for risk in the model. The change from homogeneous capital to heterogeneous capital has more implications than just adding a sector-specific subscript. In models of this type, heterogeneous capital can introduce exogenous risk premia.

The market rental rate of capital is determined by market forces, the supply of and demand for capital. Total investment demand equals the use of investment goods from domestic and imported sources. Economy-wide, a composite investment good is derived from the final investment demand column from the input–output table. The composite investment good is allocated to sector-specific investment so that the marginal productivity of capital is equal across sectors. Investment opportunities are arbitrated when the net rate of return from each

sectorally differentiated investment does not exceed the rate of interest. When investment is undertaken in that sector the net rate of return in that sector will equal the rate of interest.

This arbitrating condition means that sectors with high gross return and lower depreciation rate generate more gross investment demand. In the steady-state, investment will grow at the same rate in all sectors, and the return to capital will be equalised across all sectors. However, during the transitional phase, it is possible for the net return in one sector to fall below that of another. As a result, investment can be shut down in the low return sector. Similar to the single-sector model, assets depreciate. Gross sectoral investment increases the capital stock as well as replaced depreciated capital.

Different from the other models, the multi-sector dynamic forward-looking model can introduce a risk premia between the different rates of return on sector-specific investment. Examples of applied models where this has been introduced can be found in Section 3.2.1 of this paper. Nevertheless, this neoclassical risk is exogenous.

Based on these limitations it might be argued that there is no room for risk to be incorporated into a CGE model. The CGE methodology does not allow it. The next section reviews a selection of instances where elements of risk have been incorporated into applied models.

### 3. Review of CGE models incorporating risk

There have been several attempts in the literature to make explicit allowance for risk and uncertainty in CGE models. We can categorise these in two ways: first, in terms of the type or source of risk and

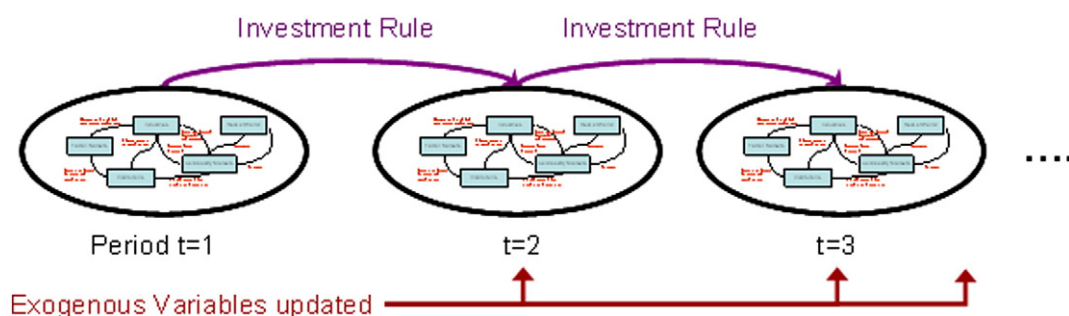


Fig. 2. Graphical representation of a dynamic recursive model.



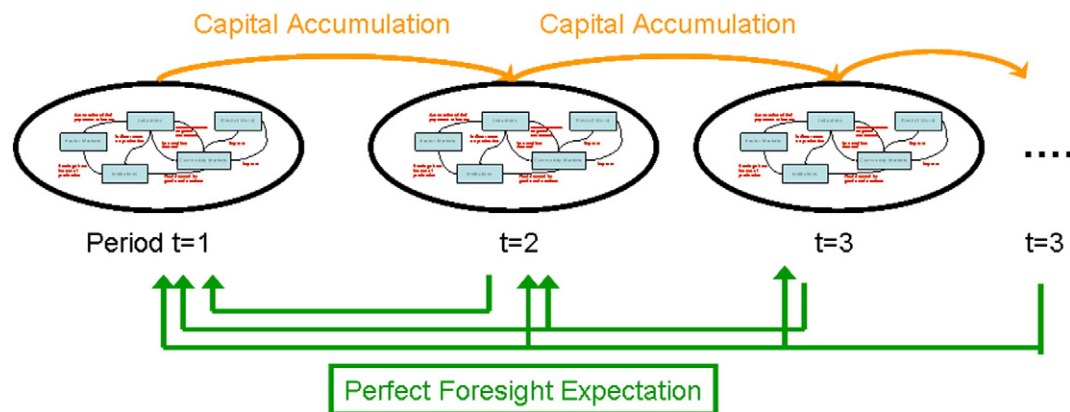


Fig. 3. Graphical representation of a dynamic forward-looking model: infinite horizon.

uncertainty; second, in terms of the method(s) by which risk is handled within the CGE model.

### 3.1. Model uncertainty

Model uncertainty relates to the risk of reporting incorrect results, the uncertainty about the true value of exogenous parameters used in the model, and the model specification. For example, the modelling process may involve uncertainty about true values of elasticities. A CGE model relating to a particular region or a particular time period may actually use elasticity estimates that were econometrically estimated using data that relate to different regions or different time periods. In short, there is a mismatch between the data sample and the source of variation used for econometric estimation, and the policy experiment explored in the CGE model (Hertel et al., 2004). A good way to account for this type of uncertainty in CGE models is the systematic use of sensitivity analysis – specifically, Monte Carlo analysis or the Gaussian quadrature procedure (DeVuyst & Preckel, 1997). The model equations use the constant elasticity of substitution (CES) family of equations. For example the Leontief production function occurs when the elasticity of substitution approaches negative infinity and, using l'Hospital's rule, it can be shown that CES function reduces to a Cobb–Douglas when, in the limit, the elasticity of substitution is equal to zero.

Modellers may also face uncertainty about the reliability of results obtained from their CGE model simulations. In general, there is a risk of reporting inaccurate results. In particular, when the source data for CGE models are input–output tables and Social Accounting Matrices (SAMs) relating to a particular benchmark year, the assumed production functions and consumer preference functions are determined by a process of calibration, rather than being estimated econometrically. As a result, the conventional econometric t-ratios and confidence intervals that provide a measure of reliability do not exist, and hence the modeller faces uncertainty about the accuracy of results and any margin of error. Some studies attempt to validate results through econometric techniques. For example, Valenzuela et al. (2005), attempt to validate results from their global CGE model (GTAP), use stochastic simulation to reflect random variability in wheat production. Another study, by Gehlhar (1997), uses backcasting simulation to evaluate the validity of GTAP model results as compared to observed outcomes for East Asian economic growth in the 1980s. A further study, by Liu et al. (2004) builds on Gehlhar's approach, and develops an approximate likelihood function to assess the quality of model performance over a 6 year period.

### 3.2. Economic risk

Uncertainty about the state and future of the economy can take several forms. These include uncertainty about future events, uncertainty

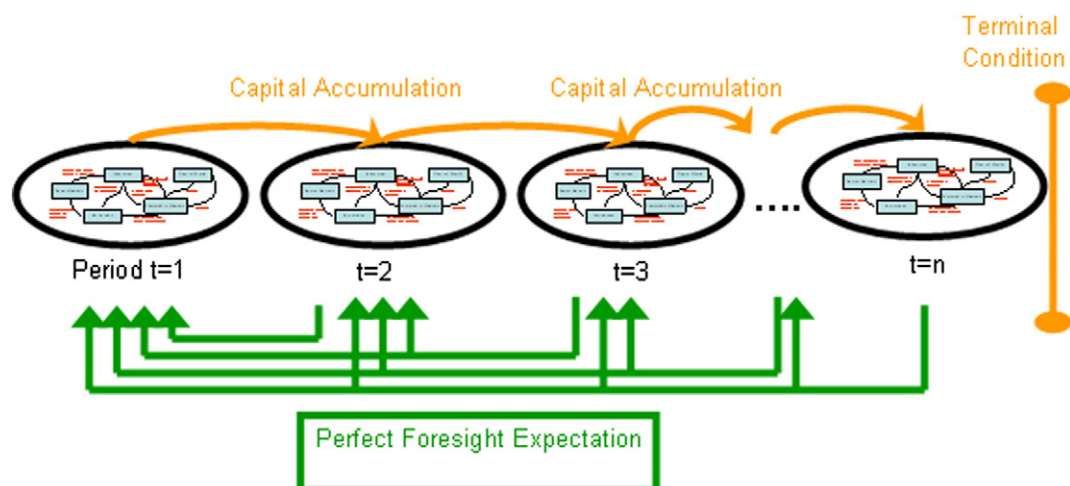


Fig. 4. Graphical representation of a dynamic forward-looking model: finite horizon.

about policy interventions, and country or technology risks — and these different categories can overlap. Here we give a brief summary of some studies that have considered these different forms of uncertainty.

### 3.2.1. Uncertainty about the future

Uncertainty can be treated as incomplete information, and hence the implications of uncertainty can be treated as a form of market inefficiency. The lack of information regarding the future may give producers an incentive to supply too much of some products and too little of others. Alternatively, consumers may not purchase a product even though they would benefit from doing so. One method of simulating incomplete information has been to contrast static expectations (incomplete information) with rational expectations (perfect information). In the first case (static expectations), consumers and producers have full information about the current time period but know nothing of the future, while in the second case (rational expectations) consumers and producers have perfect knowledge of both current and future market conditions.

One study which makes explicit allowance for uncertainty about the future information within a CGE model is the work of [Arndt and Bacou \(2000\)](#). Taking a relatively standard CGE model of Mozambique, these authors explore the value of climate forecast information that operates and interacts at a farm level, at a marketing system level and at a full economy level.

Another such study is the work of [Ianchovichina et al. \(1999\)](#) which focuses on international capital mobility. These authors develop a disequilibrium approach for a dynamic multi-sector multi-region general equilibrium model. A key feature of this model is that there are errors in investors' assessment of potential returns to capital. They argue that investors' expectations are sticky and that when the observed rates of return change, investors are uncertain whether this change is temporary or permanent. Investor expectations are only adjusted with a lag. Initially, investors make small adjustments, and if the change in the rate of return persists, they make further changes in their expectations until the expected rate equals the observed rate of return. These authors show that this feature was part of the explanation for the Asian financial crisis of 1997.

The work of [Ianchovichina et al. \(1999\)](#) also makes use of exogenous risk premia to reflect some of the uncertainties within their model. This is a technique developed further by [McKibbin and Wilcoxon \(1998, 1999\)](#). In their G-Cubed model, [McKibbin and Wilcoxon \(1999\)](#) incorporate exogenous exchange rate risk premia in their inter-temporal and dynamic multi-sector, multi-region CGE model. They do this through the full integration of real and financial markets. Their assumption of perfectly integrated asset markets across regions implies that the expected returns on loans (interest rates plus risk premia), denominated in one region (currency), is equal to the expected returns in another region (currency) adjusted by the exchange rate. Within each economy, the expected returns to each type of asset are equalised by arbitrage, adjusting for adjustment costs of physical capital stock and exogenous risk premia. In long run equilibrium, the return to capital is the same across different sectors, but in the short run, simulations can allow for arbitrage and hence risk premia across different capital assets.

### 3.2.2. Policy uncertainty

Along the same lines as [Arndt and Bacou \(2000\)](#), but in a different context, [Adams et al. \(2001\)](#) make allowances for uncertainty about the timing and announcement of policy changes within a dynamic CGE model. They model two scenarios: first, the introduction of a once-off quota for pig production in the Danish economy, without any previous announcement; and second, a production quota that is announced, and phased in gradually. Not surprisingly, the adjustment path of the economy is smoother when the policy is announced that when the policy is implemented as a surprise.

[Boussard et al. \(2002\)](#) examine the uncertainty around agricultural trade liberalisation, while also making allowance for imperfect

expectations. They model this instability by endogenising risk through lags in delivery, and making allowance for risk aversion. Uncertainty is incorporated in the model through a production lag in the agricultural sector. Building on the work of [Ezekiel \(1938\)](#), who developed the cobweb theorem, [Boussard et al.](#) assume a lag between production and consumption decisions in the agricultural sector. The market equilibrium is defined with reference to the previous year's production and the current year's consumption. In contrast to the classical perfect foresight model, where global gains are associated with trade liberalisation, [Boussard et al.](#) find that the model with risk aversion, imperfect information and an agricultural production lag produces negative changes in real income. Imperfect information constrains the economy from reaching its optimum.

In the CGE literature, a common way to handle incomplete information about the future is by using a dynamic recursive model. The dynamic recursive model has some inherent problems, however. The researchers cited above argue that a dynamic forward-looking model does not permit imperfect information or errors in expectations. But, on the other hand, to argue that economic agents do not base decisions on what they know about the future, nor do they attempt smooth consumption or production, is surely to err in the opposite direction. In the next section, we outline an alternative method that treats uncertainty and risk explicitly by considering multiple future paths for the model. This approach assumes that agents can predict each path, and make decisions in the presence of this uncertainty.

[Boussard et al. \(2002\)](#), allowing for imperfect expectations, examine the issue of agricultural trade liberalisation, adding instability in the model by endogenising risk through lags in delivery, and risk aversion. Uncertainty is introduced into the model through a production lag in the agricultural sector. Picking up the work done by [Ezekiel \(1938\)](#), who developed cobweb theorem, the researchers specify a lag between the production and consumption decision for the agricultural sector. The market equilibrium occurs between the previous year's production and the current year's consumption. They find, in contrast to the classical perfect foresight model where global gains are associated with trade liberalisation, the model with risk aversion, imperfect information and a production lag in the agricultural sector shows negative changes in real income. Imperfect information constrains the economy from reaching its optimum.

### 3.2.3. Country and technology risk

In a similar way, country risk can be modelled by using exogenous risk premia in a dynamic CGE model ([Malcolm, 1998](#)). The standard GTAP model assumes that the global bank equalises risk-adjusted rates of return so that the risk-adjusted rates for all regions are equal to a weighted average of returns around the world. [Malcolm \(1998\)](#) defines these risk premia explicitly, and proceeds to examine the effects of changes in these risk premia. It is worth stressing that, in these multi-sector models, such risk premia are exogenous.

Another method of making risk endogenous with a CGE model has been developed by [Arndt and Tarp \(2000\)](#). They employ a CGE model to analyse the interactions between agricultural technology improvement, risk, and gender roles within the agricultural sector in Mozambique. They introduce a particular type of 'technology risk' into their model, by assuming that a safety first strategy is pursued. That is, they assume that households aim to produce a given amount of cassava (the crop of interest in their study) for risk reduction purposes only. [Arndt and Tarp](#) conclude that there are considerable differences in production and price movements for cassava between the 'risk' and 'no risk' scenarios.

In terms of the type of uncertainty modelled in this paper and described in the next section, the locus of uncertainty is within the economy, the source of uncertainty is related to the uncertainty of future events and hence expectations and the method of treatment in CGE modelling is the contrast between static and rational expectations.

#### 4. Uncertainty regarding future paths of the economy: conceptual framework

This section outlines how risk can be incorporated into a dynamic forward-looking CGE model through the uncertainty of the future path of the economy. The general equilibrium framework has been developed to economic situations involving the exchange and allocation of resources under conditions of uncertainty (Mas-Colell et al., 1995). In these theoretical models, the concept of uncertainty is formalised by means of different states of the world. That is, technologies, endowments, and preferences depend on the state of world. A contingent commodity is a commodity whose delivery is conditional on the realised state of the world. This “state-preference” approach to uncertainty, introduced by Arrow (1964) and further detailed by Debreu (1959, Chapter 7) assumes that there is a market for every contingent commodity. The introduction of contingent commodities then sees the concept of a Walrasian equilibrium become an Arrow–Debreu equilibrium. Arrow–Debreu equilibrium results in a Pareto optimal allocation of risk. The Arrow–Debreu framework then moves into the creation of spot markets and forward markets, arriving at the Radner equilibrium (1972) where economic agents form expectations of spot prices in future states, purchase present goods and securities on the basis of those expectations. Current and future spot prices of goods and assets adjust so that all “markets” clear and these price expectations must be fulfilled. This is now the foundation of finance theory (for an introduction see Duffie, 1992; Huang & Litzenberger, 1988).

Applied CGE models have also integrated financial flows and assets with the neoclassical CGE model. Robinson surveys these ‘micro-macro’ CGE models that incorporate asset markets and product and factor markets (Robinson, 1991). The distinguishing feature of these models is “in their specification of the loanable funds markets, with a variety of different assets including currency, demand deposits, time deposits, government debt, domestic bonds, foreign bonds, equity real capital, and working capital.” (Robinson, 1991 p.1517). Typically, the underlying SAM will disaggregate the capital account to include different types of assets from the different economic agents.

The model outlined in this thesis does not include a loanable funds market but several features of the theoretical Arrow–Debreu framework are implemented. Uncertainty in the model means different states of the world or paths that the economy might take. Due to the different states of the world, risk is created but the uncertainty of which paths will occur — one or more paths will contain a shock which may be realised. An exogenous variable, for example tourism demand, can be simulated to vary on any or all of the possible paths the economy might take. Economic agents have perfect sight across all the possible paths the economy might take. The assumption of model-consistent rational expectations will hold so that the representative consumer, firms, and government are endowed with perfect foresight, and so anticipate any exogenous shocks and adjust their maximising behaviour from the second time period (the period in which the uncertainty, but not necessarily the shock, will occur).

Given this uncertainty, how do consumers and firms change their behaviour? For example, in the partial equilibrium literature, firms will under-invest in sectors where there is uncertainty (Bernanke, 1983; Cuikerman, 1980; Hartman, 1972; Majd & Pindyck, 1987; Marschak, 1949; Pindyck, 1982). Do these results hold in the general equilibrium context? The economic impacts of the timing (when the shock occurs and how long the shock lasts) as well as the magnitude of the exogenous shocks will also be analysed in the next section.

A graphical representation of the conceptual model is shown in Fig. 5. In this example, two paths are possible. One path follows the baseline growth, while the other path experiences a policy shock. These impacts are then followed through to the  $n$ th time period. Expectations are consistent throughout so that economic agents have lower expectations under the negative shock path from the first

time period, even though the negative shock does not take place until period  $t = 2$ . Further, the probability that a path may take is set objectively, as an exogenous parameter. In the model above (Fig. 5), the benchmark case (P1) is assumed to continue with a probability of 80% and the case with the negative policy shock is assumed to occur with a probability of 20%. Policy makers can use the model to investigate the economic impact of a shock to the economy with a particular certainty.

In the initial time period,  $t = 0$ , the model solves and is calibrated as if it were a comparative static model. In period  $t = 1$ , there is uncertainty on which path the economy might follow. The next step is to introduce a number of different paths that the economy might take as well as the probability that each path might take along a certain path. Let  $p$  be the number of possible path, then  $\Phi(p)$  is the probability that

a specific path is taken. It is a necessary condition that  $\sum_{p=1}^P \Phi(p)$ , that

is, that the probability of the sum of the paths sum to 1. To calibrate the model: from the second time period onwards ( $t = 1$ ), all the sectors and benchmark quantities need to be multiplied by the probability that this particular path occurs. The standard economic relationships exist for the first time period and in each path,  $p$ . The capital accumulation links the  $t = 0$  no-uncertainty part of the model to the dynamic uncertainty part of the model ( $t \geq 1$ ). Hence, where  $K^*$  indicates the first time period and  $K_{p,t}$  indicates capital in period  $t$  for path,  $p$ . The Ramsey economic growth dynamics (Ramsey, 1928); the capital accumulation equation; now becomes:

$$K_{p,t=1} = K^*(1-\delta) + I^* \quad (1)$$

$$K_{p,t+1} = K_{p,t}(1-\delta) + I_{p,t} \quad (2)$$

where  $*$  in Eq. (1) denotes first-year values ( $t = 0$ ) and Eq. (2) represents the capital accumulation equation from the second period onwards ( $t \geq 1$ ).

Production can be decomposed in this model for the time period  $t = 0$  and for  $t \geq 1$ . As in a standard CGE model, in the initial period sectors  $i$ 's production function is  $Y_i = g(D_i E_i) = f(K_i L_i A_{ij})$ . For the dynamic component of the model, sector  $i$ 's production function is dependent on time and the path the economy takes hence:  $Y_{i,t,p} = g(D_{i,t,p} E_{i,t,p}) = f(K_{i,t,p} L_{i,t,p} A_{ij,t,p})$  where  $g$  is output transformation function and  $f$  is input transformation function. Specifically, the initial output transformation takes the form of a constant elasticity of transformation

$$(CET): Y_i = \theta \left( \delta_i^e D_i^{\frac{\eta-1}{\eta}} + (1-\delta_i^e) E_i^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}} \text{ and the multi-path dynamic}$$

$$\text{output is expressed as } Y_{i,t,p} = \theta \left( \delta_{i,t,p}^e D_{i,t,p}^{\frac{\eta-1}{\eta}} + (1-\delta_{i,t,p}^e) E_{i,t,p}^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}} \text{ where}$$

$Y$  = output;  $E$  = exports;  $D$  = domestic production;  $\eta$  = the elasticity of transformation in total supply;  $\delta_{i,t,p}^e$  = the calibrated share of exports; and  $\theta$  = the calibrated shift parameter in the transformation function.

Similarly, the Armington aggregate of domestic output and imports in the initial time period is specified as a constant elasticity of substitution (CES) function:

$$A_i = \Omega \left( \delta_i^m D_i^{\frac{\gamma-1}{\gamma}} + (1-\delta_i^m) M_i^{\frac{\gamma-1}{\gamma}} \right)^{\frac{\gamma}{\gamma-1}}$$

while the multi-path dynamic CES function of the Armington aggregate is given by  $A_{i,t,p} = \Omega \left( \delta_{i,t,p}^m D_{i,t,p}^{\frac{\gamma-1}{\gamma}} + (1-\delta_{i,t,p}^m) M_{i,t,p}^{\frac{\gamma-1}{\gamma}} \right)^{\frac{\gamma}{\gamma-1}}$  where

$A$  = the Armington CES aggregate of domestic supplies,  $D$  and imported supplies,  $M$  for each sector;  $\gamma$  = the elasticity of substitution in the aggregate supply function;  $\delta_{i,t,p}^m$  is the share of imported goods; and  $\Omega$  = the calibrated shift parameter of the aggregated supply function.

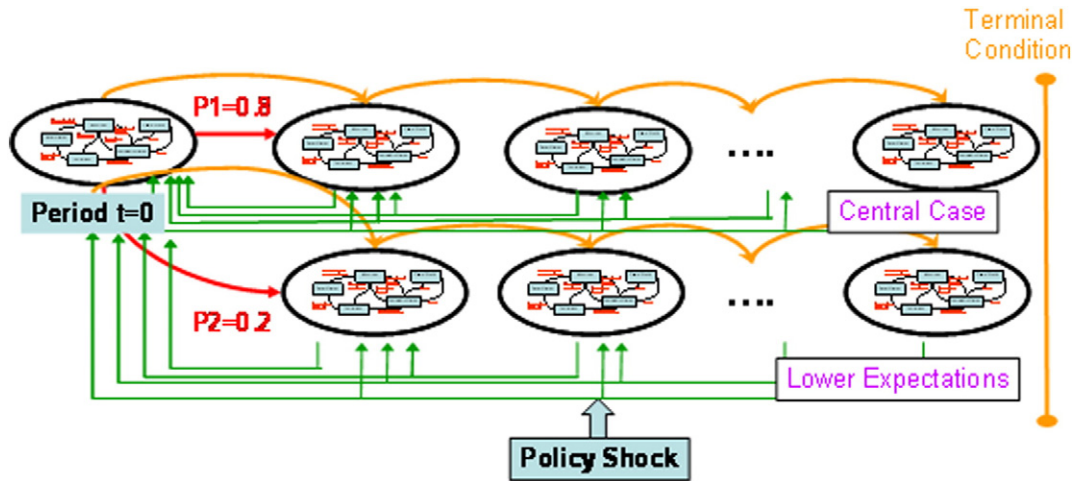


Fig. 5. Dynamic forward-looking model: with uncertainty.

As with other sections of the economy, the production of goods follows a nested Leontief–Cobb Douglas production function with the  $t = 0$  intratemporal model and the multi-path dynamic part of the model being a function of time and path. Hence, in the initial period,  $t = 0$ , output is allocated to the domestic and export markets according to a constant-elasticity-of-transformation and intermediate inputs are Leontief, while labour and capital enter as a Cobb–Douglas value-added aggregate:

$$f(K_i, L_i, A_{ij}) = \min \left\{ B_i L_i^{\alpha_i} K_i^{(1-\alpha_i)}, \min \left\{ \frac{A_{i,1}}{a_{i,1}}, \frac{A_{i,2}}{a_{i,2}}, \dots, \frac{A_{i,j}}{a_{i,j}} \right\} \right\}.$$

In the dynamic section of the model, from  $t \geq 1$ , the production of goods is denoted by a nested Leontief–Cobb Douglas production function in every time period and on every path:

$$f(K_{i,t,p}, L_{i,t,p}, A_{i,j,t,p}) = \min \left\{ B_{i,t,p} L_{i,t,p}^{\alpha_i} K_{i,t,p}^{(1-\alpha_i)}, \min \left\{ \frac{A_{i,j,t,p}}{a_{i,j,t,p}} \right\} \right\}.$$

The representative household maximises the present value of their lifetime utility across paths. Utility, now dependent on the time path chosen, is represented by:

$$U_p = \sum_{t=1}^{\infty} \left( \frac{1}{1+\rho} \right)^t c_{p,t}. \quad (3)$$

Total utility across all paths is given by

$$U^* = \left[ \sum_1^p \Phi(p) U_p^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \quad (4)$$

where

- $U^*$  is total utility
- $C_{t,p}$  is consumption in each time period and on each path
- $\rho$  is the discount factor or individual time-preference parameter
- $U_p$  is discounted utility on each path
- $\Phi(p)$  is the probability of each path occurring, and
- $\sigma$  is the risk aversion parameter.

As shown in Fig. 6, utility is a nested function across paths and across time. At the top level, total utility is a composition of utility in each path with the elasticity of substitution between paths – the

risk aversion parameter denoted by  $\sigma$ , as in Eq. (4). At the next level down, utility on path,  $p$ , is a composite of utility in each time period with the elasticity of substitution between time periods denoted by  $\rho$ .

Welfare is measured by the equivalent variation metric (EV). EV at time period 0 is given by:  $\frac{C_1 - C_0}{C_0}$  where  $C_1$  is consumption after the counterfactual;  $C_0$  is consumption in the benchmark (normalised to 1) and;  $C_0$  is the benchmark level of consumption. EV in each path is given by  $EV(p) = \frac{UP_1(p) - UP_0(p)}{UP_0(p)} \cdot \sum_t (QRef(t) \cdot PRef(t) \cdot C_0)$  where  $UP_1(p)$  is utility after the counterfactual;  $UP_0(p)$  is utility in the benchmark (normalised to 1);  $QRef(t)$  is the reference growth path for the economy given by  $(1 + \gamma)^t$ ;  $PRef(t)$  is the reference growth path for prices in the economy given by  $PRef(t) = \left[ \frac{(1-R_0)}{(1+R_0-\delta)} \right]^t$  and  $C_0$  is the benchmark level of consumption. Due to the initial time period ( $t = 0$ ),  $PRef(t)$  in the risk model is advanced one time period (raised to the power  $t$ , rather than raised to the power  $t - 1$ ). EV for the whole model is given by  $EV = \frac{U_1 - U_0}{U_0} \cdot \sum_p \Phi(p) \cdot U^*$ .

The demand shock is incorporated through exogenous tourism demand. Tourism is modelled in the following way: a representative tourism household demands tourism (a certain quantity of a composite good and service) at an aggregated tourism price level,  $PT$ . In the benchmark, tourists are aggregated so there is a representative tourist accounting for all tourists' consumption. Tourism demand is obtained by maximising the utility function of the representative tourist subject to their budget constraint. A constant elasticity of demand function is used whereby demand varies according to the price of the appropriate bundle of tourism goods and services; hence, Hawaii faces a downward sloping demand curve for its tourism. Tourism consumption,  $TC$ , is related to a

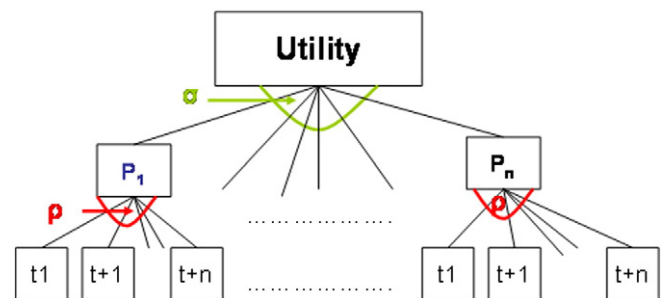


Fig. 6. Multiple path utility schematic.



composite tourism price (akin to a tourism CPI),  $PT$ , and the exchange rate,  $PFX$ , in the following manner:

$$TC_t = \overline{TC} \left( \frac{PT_t}{PFX_t} \right)^\varsigma + T^*$$

where  $\overline{TC}$  is the base level of tourism consumption,  $\varsigma$  is the price elasticity of demand for foreign tourism ( $\varsigma < 0$ ) (set at 0.5) and  $T^*$  is an exogenous parameter, set to zero in the benchmark, but can be set to different levels to model changes in tourism demand. Tourists are endowed with foreign exchange. Tourism consumption is composed of the consumption of different commodities, with a Cobb–Douglas function determining how tourists substitute between commodities. The utility of the representative tourist is a Cobb–Douglas function of consumption of the composite goods

$$TC = \varphi \prod_i tci_{i,t}^\sigma$$

where  $TC$  = aggregate tourism consumption;  $\varphi$  = a shift parameter that is calibrated to ensure the model replicates the benchmark;  $\sigma$  = the share of commodity  $i$  in tourism consumption;  $tci$  = consumption by sector;

$$tci_t = X \left( \delta_i^{tc} TCD_i^{\frac{\gamma-1}{\gamma}} + (1-\delta_i^{tc}) TCM_{i,t}^{\frac{\gamma-1}{\gamma}} \right)^{\frac{\gamma}{\gamma-1}}$$

$TCM_{i,t}$  = imported production of a tourism consumption good;  $TCD_i$ ,  $t$  = domestic production of tourism consumption good;  $\gamma$  = the elasticity of substitution between domestic goods and services and imported goods and services;  $\delta_i^{tc}$  = the calibrated share of consumed tourism domestic goods; and  $X$  = the calibrated shift parameter in the substitution function.

The benchmark data used for this study comes from the 2002 Hawaii input-output table published by the State of Hawaii's Department of Business, Economic Development and Tourism. The data set was aggregated into a twenty sector model. For the year 2002, the GDP of Hawaii was \$US 43.48 billion and tourists to Hawaii were estimated to spend \$US 9.47 billion. The modelling software used is GAMS (Brooke et al., 1988) with an MPSGE subsystem (Rutherford, 1999) and solved using the PATH solver.

## 5. Uncertainty regarding future paths of the economy: results

The model time horizon will also be the same but by specifying different paths the economy can take, the nomenclature will be different. In a standard dynamic forward-looking model across 50 time periods, the model runs from  $t = 1$  to  $t = 50$ . In a multi-path dynamic forward-looking model, the 'first' period calibrates the model and does not contain the option of multi-paths. This 'first' period is modelled like a comparative static model. Like the standard dynamic forward-looking version, the accumulation of capital links this 'comparative static' model to the ensuing multi-path dynamic model. For comparability, the last time period, will be one period less than the standard model. In the above example the dynamic part of the multi-path model would be  $t = 49$  time periods. As a check the standard  $t = 50$  period model was benchmarked against the static  $t = 0$ , multi-path dynamic  $t = 49$  model and the benchmark values were equivalent.

Operationalising this CGE model with risk, a natural counterfactual would be to assume the economy continues on the business as usual growth path with a probability of 50% (path 1) and to model a 10% decrease in tourism demand from the 9th time period onwards with a probability of 50% (path 2). This counterfactual is shown pictorially in Fig. 7. The value of the risk aversion parameter,  $\sigma$ , has been set to 0.5. This specification ( $\sigma < 1$ ) implies the representative household is risk averse. When  $\sigma = 1$ , the representative household is risk neutral and when the  $\sigma > 1$ , the representative household is risk seeking. The elasticity between time periods: time preference elasticity,  $\rho$  has been set to 1, implying the representative household's utility is a Cobb–Douglas function (fixed proportions) across time.

In a model of this sort, welfare can be decomposed into welfare from path 1, welfare from path 2 and welfare from the initial time period ( $t = 0$ ). The sum of these three components will not usually equal total welfare across the model due to the non-linear nature of the model. The result tables will display the decomposed welfare changes as well as the overall change in welfare.

Table 1 shows the decomposition of several variables across the three time dimensions: the initial period ( $t = 0$ ), path 1 and path 2. The rows in the table are equivalent variation (EV) in \$US million, the percentage change in EV, the terms of trade in \$US million (this shows the amount changed due to the change in prices), the amount

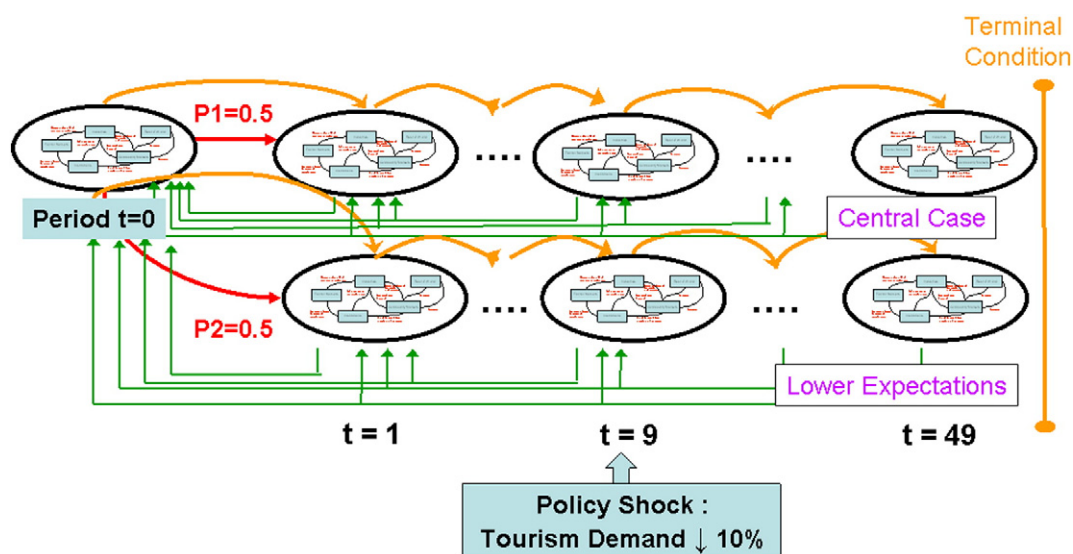


Fig. 7. Possibility of a negative 10% tourism demand shock from  $t = 9$  onwards.

**Table 1**Results: 2 path 10% negative shock on 1 path in  $t = 9$  onwards.

Time period	$t = 0$	Path 1	Path 2	Total
EV	\$150.3 m	\$372.4 m	–\$3713.5 m	–\$1537.7 m
% EV (% change from Benchmark)	0.66%	0.13%	–1.26%	–0.48%
Terms of trade	\$5.1 m	–\$2587.3 m	–\$17,143.3 m	–\$9860.2 m
Tourism demand shift	–	–	–\$84,337.9 m	–
Tourism price (% change from Benchmark)	0.045%	–0.24%	–1.79%	–

in \$US million of the tourism demand shift being simulated and the percentage change in the tourism price.

The table shows some interesting results. Firstly, the consumers start adjusting their behaviour from the period  $t = 0$ , that is, even before the model splits into the two paths. This observation is standard in rational expectation models and should be expected but it is significant nonetheless as consumer behaviour of this kind marks a significant departure from the consumer behaviour in dynamic cursive models. Secondly, in the path where there is no tourism demand shock, travelling along this path increases welfare. Along this path, welfare increases by \$US 372.4 million across the model horizon, equating to a slight increase in welfare of 0.13%. As expected, welfare in path 2, where the simulated shock occurs decreases by 1.26%. In total, welfare decreases by 0.48%. This stands in contrast to the definitive case where tourism demand is simulated to fall 10% with certainty from  $t = 10$  onwards (the equivalent of  $t = 9$  in the risk model), which reported a decrease of 1.63% in welfare.

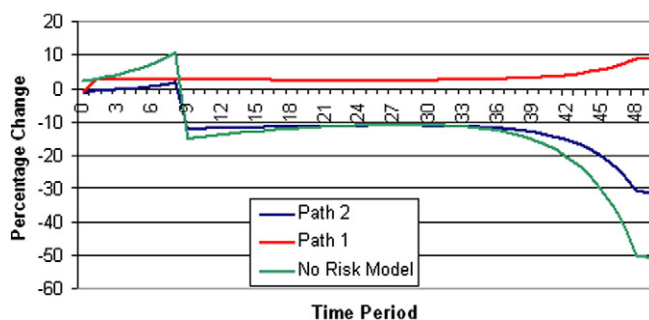
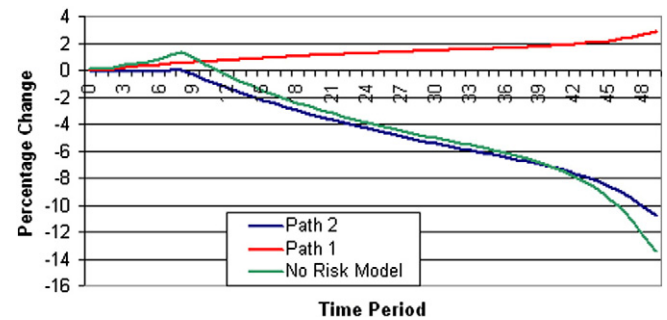
Figs. 8 to 11 show the transition paths of investment, capital, residents' consumption and tourism consumption respectively for the no-risk model, and paths 1 and 2 for the risk model for a 10% decrease in tourism demand starting from period  $t = 9$  onwards.

Investment, capital and residents' consumption on path 1 (the path with no counterfactual) grow at a positive rate above the baseline benchmark growth rate. Path 2 investment does not fall as far as investment in the 'no-risk model'. This is to be expected as there is only a 50% probability of a tourism demand shock along path 2.

Further, with the counterfactual on path 2 being only likely to occur with a probability of 50%, the percentage change in investment, capital, resident's consumption and tourism consumption is only a proportion of 'no-risk' model rate.

A second scenario of interest would be to model both a hypothetical tourism boom and tourism bust with the same probability occurring in the same time period. This scenario is shown pictorially in Fig. 12.

Table 2 shows that the absolute value of the equivalent variation from path 1 is greater than the absolute value of the equivalent variation from path 2. This is because the tourism price in path 1 increases more than the tourism price in path 2 decreases. This is due to the risk aversion of the representative consumer. The increase in EV for the total model is \$US 2.7 million, again reflecting the non-linear nature

**Fig. 8.** Investment in the risk vs. no-risk model.**Fig. 9.** Capital in the risk vs. no-risk model.

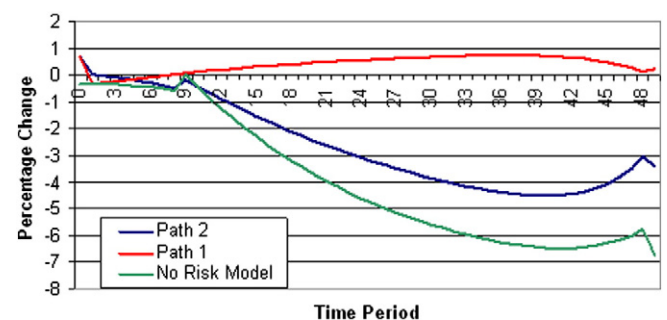
of the model – the negative shock is not the equivalent of the opposite of a positive shock. The difference in the absolute value of path 1 and path 2's equivalent variation can be interpreted as the value of risk – in this scenario – it is worth \$US 166.6 million.

This section summarises the economic impact of uncertain tourism demand by incorporating risk in a dynamic forward-looking CGE model. The possibility of a negative tourism demand shock induces welfare losses. While the economy might take many trajectories, for tractability this research primarily examines a two-path model. In the scenario where there is an asymmetric shock (50% probability of benchmark growth on path 1, 50% probability of a 10% negative tourism demand shock on path 2), on the non-shocked path, there are welfare gains while, as expected, on the shocked path welfare decreases. The percentage change in overall welfare across all paths is negative. The non-shocked path experiences welfare gains as investment and capital increases at a positive rate on this trajectory, above the benchmark where the representative household is assumed to be risk averse.

In a two-path model with symmetric shocks (50% probability of a 10% tourism boom and 50% probability of a 10% tourism bust), the total welfare increases marginally by \$US 2.7 million but the welfare gain on the tourism boom path is larger than the welfare loss on the tourism bust path. This is due to the risk aversion characteristic of the representative household. Further, the difference in the absolute value between the welfare gain and the welfare loss on the two paths can be interpreted as the cost of the information regarding which path economy will travel. Another way to look at it would be the cost of the uncertainty or the monetary value of the risk in the model.

## 6. Conclusions and further research

CGE models have traditionally tended to be deterministic in nature. In this paper, a way to model risk and uncertainty in a CGE model has been demonstrated. The risk is incorporated into the model through the introduction of uncertainty regarding the future

**Fig. 10.** Consumption in the risk vs. no-risk model.

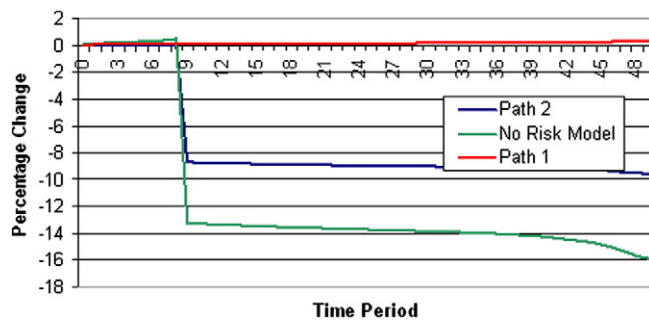


Fig. 11. Tourism consumption in the risk vs. no-risk model.

path of the economy. The probability and timing of a shock to the model economy is the underlying source of uncertainty.

Various scenarios have been simulated. For example, one what-if scenario was to model a 50% probability of 10% negative tourism demand shock from time period  $t = 9$  until the end of the model horizon along with a 50% probability that the economy would continue along its usual growth path. In this scenario, the expected value of welfare decreases by \$US 1537.7 million or 0.48%. If the risk is realised and shock occurs, welfare decreases by \$US 3713.5 million or 1.26%. If the economy were to follow the non-shocked path, households would receive \$US 372.4 million in welfare or 0.13%, above the baseline. The welfare gains along the non-shocked path are a result of household's risk aversion and their substituting resources away from the shocked path. The difference in the monetary values of the welfare on either path can be interpreted as the 'price' of the risk. It is the price households would be willing to pay to guarantee no tourism shock. Another scenario was to model a 50% probability of 10% negative tourism demand shock from the time period  $t = 9$  until the end of the model horizon in conjunction with a 50% probability that the economy would experience a positive tourism demand shock from the same point in time. The expected value of welfare was marginally positive meaning the welfare gain from the tourism boom is greater

Table 2

Results: 2 path 10% positive & negative shock on both paths from  $t = 9$  onwards.

Time period	$t = 0$	Path 1	Path 2	Total
EV	–\$19.0 m	\$4343.0 m	–\$4176.5 m	\$2.7 m
% EV (% change from Benchmark)	–0.08%	1.47%	–1.42%	0.00%
Terms of trade	–\$0.9 m	\$15,270.9 m	–\$14,465.8 m	\$401.7 m
Tourism demand shift	–	\$93,708.7 m	–\$84,337.9 m	–
Tourism price (% change from Benchmark)	–0.008%	1.62%	–1.54%	–

than the welfare loss from the tourism bust. This value can also be interpreted as the cost of imperfect information.

There are more avenues that could be explored using this type of analysis. The analysis of tourism demand is only one area where an exogenous shock can be modelled with risk due to external factors such as global political and health situations. The area of agricultural economics lends itself to this type of analysis. For example, the introduction of cash crops in an economy reduces poverty and is generally seen as welfare enhancing but, due to the vagaries of climate and weather, agriculture can be a riskier activity than other sectors. Noting the inherent uncertainty in the weather and the implications it has for the agricultural sector, leads questions of how this type of modelling might be used to model climate change, where uncertainties about future impacts of climate change can be included in a model to show the effects of this uncertainty. Another interesting branch of research could be to investigate what other policy actions the government might do, if anything, to decrease the amount of uncertainty in order to increase long term growth in the economy.

This paper makes original contributions in the literature both methodologically and notes policy implications as a result of the inclusion of risk in the model. The research investigates the economic impact of uncertain tourism demand. The method used in this paper evaluates the influence of unanticipated shocks through the uncertainty of the future path of the economy. This imperfect information results in a market distortion. As such, there may be a suboptimal level of tourism production and a welfare loss. One policy implication for government as a result of

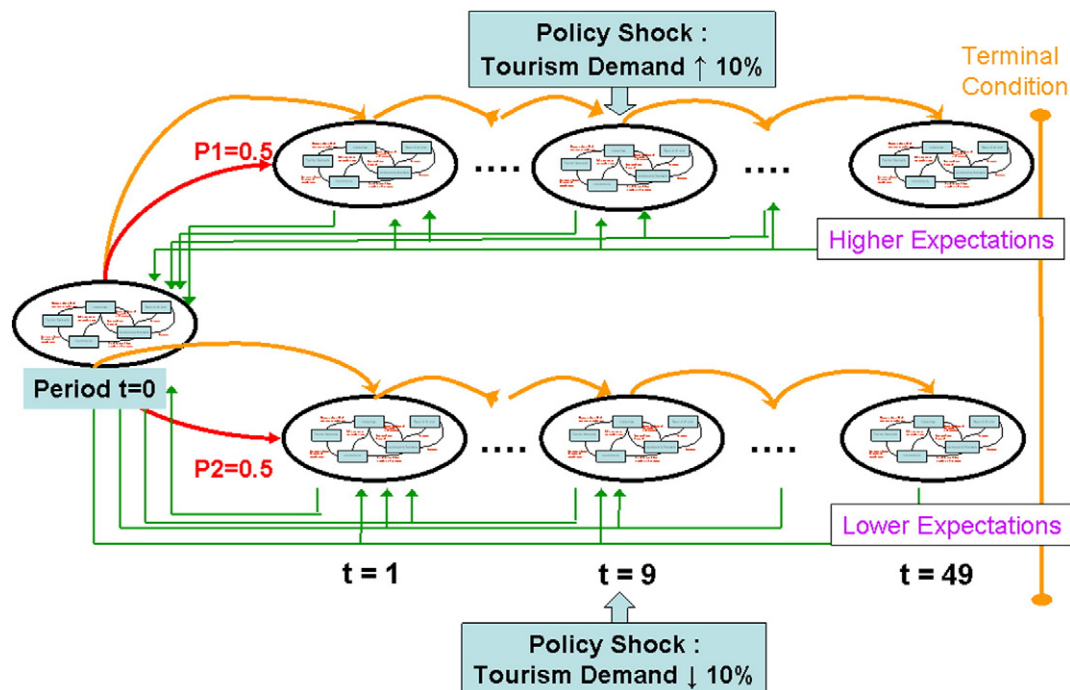


Fig. 12. Possibility of a positive & negative tourism demand shock from  $t = 9$  onwards.

this imperfect information could be the use of tourism taxes or credits to offset the loss of income due to the uncertainty of future tourism demand. The imposition of an additionally tourism tax would generate tax revenues, which would eventually be distributed back to residential households by government. Hence, the revenue generated by the additional taxes would need to be as great as the income lost as a result of the tourism bust – an example of the theory of second best.

## References

- Adams, P.D., Hansen, L.T., Jacobsen, L.-B., 2001. Does timing and announcement matter? Restricting the Production of Pigs within a Dynamic CGE Model. : Fourth Annual Conference on Global Economic Analysis. Purdue University, West Lafayette, Indiana, USA.
- Anscombe, F.J., Aumann, R.J., 1963. A definition of subjective probability. *Annals of Mathematical Statistics* 34, 199–205.
- Arndt, C., Bacou, M., 2000. Economy-wide effects of climate variation and climate prediction in Mozambique. *American Journal of Agricultural Economics* 82 (3), 750–754.
- Arndt, C., Tarp, F., 2000. Agricultural technology, risk, and gender: a CGE analysis of Mozambique. *World Development* 28 (7), 1307–1326.
- Arrow, K., 1964. The role of securities in the optimal allocation of risk-bearing. *Review of Economic Studies* 31, 91–96.
- Arrow, K., 1965. Aspects of the Theory of Risk Bearing. Yrjö Jahnssonin Saatio, Helsinki.
- Arrow, K., Debreu, G., 1954. Existence of and equilibrium for a competitive economy. *Econometrica* 22 (3), 265–290.
- Barro, R.J., Sala-i-Martin, X., 1995. *Economic Growth*. McGraw-Hill Inc., New York.
- Bernanke, B.S., 1983. Irreversibility, uncertainty, and cyclical investment. *Quarterly Journal of Economics* 98 (1), 85–106.
- Bluhm, C., Overbeck, L., Wagner, C., 2002. *An Introduction to Credit Risk Modeling*. Chapman & Hall/CRC.
- Boussard, J.-M., Gerard, F., Piketty, M.G., Christensen, A.K., Voituriez, T., 2002. Agricultural Trade Liberalisation in a World of Uncertainty: Discussion of the Results of a World CGE Model. 5th Annual Conference on Global Economic Analysis (Taipei). Retrieved May 5 2006, from <https://www.gtap.agecon.purdue.edu/resources/download/1026.pdf>.
- Brooke, T., Kendrick, D., Meeraus, A., 1988. *GAMS: A User's Guide*. The Scientific Press, Redwood City, California.
- Cass, D., 1965. Optimal growth in an aggregate model of capital accumulation. *Review of Economic Studies* 32 (3), 233–240.
- Cuikerman, A., 1980. The effects of uncertainty on investment under risk neutrality with endogenous information. *Journal of Political Economy* 88 (3), 462–475.
- Debreu, G., 1959. *The Theory of Value*. Wiley, New York.
- Denenberg, H.S., Eilers, R.D., Melone, J.L., Zeltin, R.A., 1974. *Risk and Insurance*, 2nd ed. Prentice-Hall, New Jersey.
- DeVuyst, E.A., Preckel, P.V., 1997. Sensitivity analysis revisited: a quadrature-based approach. *Journal of Policy Modeling* 19 (2), 175–185.
- Dixon, P.B., Pearson, K.R., Picton, M.R., Rimmer, M.T., 2005. Rational expectations for large CGE models: a practical algorithm and a policy application. *Economic Modelling* 22 (6), 1001–1019.
- Duffie, D., 1992. *Dynamic Asset Pricing Theory*. Princeton University Press, Princeton, New Jersey.
- Ezekiel, M., 1938. The cobweb theorem. *Quarterly Journal of Economics* 53 (2), 255–280.
- Finucane, M.L., Holup, J.L., 2005. Psychosocial and cultural factors affecting the perceived risk of genetically modified food: an overview of the literature. *Social Science & Medicine* 60, 1603–1612.
- Gehlhar, M., 1997. Historical analysis of growth and trade patterns in the Pacific rim: an evaluation of the GTAP framework. In: Hertel, T.W. (Ed.), *Global Trade Analysis: Modeling and Applications*. Cambridge University Press, Cambridge, MA.
- Hartman, R., 1972. The effects of price and cost uncertainty on investment. *Journal of Economic Theory* 5, 258–266.
- Hertel, T.W., Hummels, D., Ivanic, M., Keeney, R., 2004. How confident can we be in CGE-based assessments of free trade agreements? NBER Working Paper No. 10477 (from <http://www.nber.org/papers/w10477>).
- Huang, C.F., Litztenberger, R., 1988. *Foundations of Financial Economics*. North-Holland, Amsterdam.
- Ianchovichina, E., McDougall, R., Hertel, T.W., 1999. A disequilibrium model of international capital mobility. Paper Presented at the Second Annual Conference on Global Economic Analysis, Denmark, June 20–22, 1999.
- Kahneman, D., Tversky, A., 1979. Prospect theory: an analysis of decisions under risk. *Econometrica* 47, 313–327.
- Kammen, D.M., Hassenzahl, D.M., 1962. *Should We Risk It: Exploring Environmental, Health, and Technological Problem Solving*. Princeton University Press, New Jersey.
- Knight, F., 1921. *Risk, Uncertainty, and Profit*. Houghton Mifflin Company, Boston.
- Koopmans, T.C., 1965. *On the Concept of Optimal Growth The Economic Approach to Development Planning*. North-Holland, Amsterdam.
- Liu, J., Arndt, C., Hertel, T.W., 2004. Parameter estimation and measures of fit in a global general equilibrium model. *Journal of Economic Integration* 19 (3), 626–649.
- Majd, S., Pindyck, R.S., 1987. Time to build, option value, and investment decisions. *Journal of Financial Economics* 18, 7–27.
- Malcolm, G., 1998. Modeling country risk and capital flows in GTAP. GTAP Technical Paper No. 13 (from <https://www.gtap.agecon.purdue.edu/resources/download/29.pdf>).
- Marschak, J., 1949. Role of liquidity under complete and incomplete information. *The American Economic Review* 39 (3), 182–195.
- Mas-Colell, A., Winston, M.D., Green, J.R., 1995. *Microeconomic Theory*. Oxford University Press.
- McKibbin, W.J., Wilcoxon, P.J., 1998. Macroeconomic volatility in general equilibrium. *Brookings Discussion Papers in International Economics*, No. 140.
- McKibbin, W.J., Wilcoxon, P.J., 1999. The theoretical and empirical structure of the G-Cubed model. *Economic Modelling* 16, 123–148.
- Pindyck, R.S., 1982. Adjustment costs, uncertainty, and the behavior of the firm. *The American Economic Review* 72, 415–427.
- Pratt, J., 1964. Risk aversion in the small and in the large. *Econometrica* 32, 122–136.
- Radner, R., 1972. Existence of equilibrium of plans, prices and price expectations in a sequence of markets. *Econometrica* 40 (2), 289–303.
- Ramsey, F.P., 1928. A mathematical theory of saving. *Economic Journal* 38 (152), 543–559.
- Robinson, S., 1991. Macroeconomics, financial variables, and computable general equilibrium models. *World Development* 19 (11), 1509–1525.
- Rothschild, M., Stiglitz, J., 1970. Increasing risk: I. A definition. *Journal of Economic Theory* 2, 225–243.
- Rothschild, M., Stiglitz, J., 1971. Increasing risk: II. Its economic consequences. *Journal of Economic Theory* 3, 66–84.
- Rutherford, T.F., 1999. Applied general equilibrium modeling with MPSGE as a GAMS subsystem: an overview of the modeling framework and syntax. *Computational Economics* 14 (1–2), 1–46.
- Rutherford, T.F., Paltsev, S., 1999. From an Input–Output Table to a General Equilibrium Model: Assessing the Excess Burden of Indirect Taxes in Russia. Retrieved May 26th, 2006, from <http://www.mpsge.org/papers/exburden.html>.
- Rutherford, T.F., 2004. Dynamic general equilibrium with GAMS/MPSGE. UNSW Workshop. (Sydney, Australia). Retrieved May 26th, 2006, from <http://www.mpsge.org/dynamics.pdf>.
- Rutherford, T.F., 2005. Geometric Sums and Terminal Approximation of the Ramsey Model. Retrieved 21 March, 2006, from <http://www.mpsge.org/termgrowth.pdf>.
- Slovic, P., 1986. Informing and educating the public about risk. *Risk Analysis* 4, 403–415.
- Slovic, P., 1987. Perception of risk. *Science* 236, 280–285.
- Slovic, P., Fischhoff, B., Lichtenstein, S., 1985. Characterizing perceived risk. In: Kates, R.W., Hohenemser, C., Kasperson, J.X. (Eds.), *Perilous Progress: Technology as Hazard*, pp. 91–123.
- Valenzuela, E., Hertel, T.W., Keeney, R., Reimer, J.J., 2005. Assessing global CGE model validity using agricultural price volatility. GTAP Working Paper, NO. 33.
- von Neumann, J., Morgenstern, O., 1944. *Theory of Games and Economic Behavior*. Princeton University Press.